## Parameter Identification of Thermal Visco-plastic Model for Magnesium Alloy AZ31D

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**ABSTRACT**: A hybrid global optimization method combining the Real-coded genetic algorithm and some classical local optimization methods is constructed and applied to develop a special program for parameter identification. The parameter identification for magnesium alloy AZ31D is carried out by using the program. A comparison of deformation test and numerical simulation shows that the parameter identification and the obtained material parameters are all available. The obtained constitutive parameters can be used for numerical simulation of hot plastic forming of the magnesium alloy parts.

**KEY WORDS**: parameter identification; dynamic recrystallization; inverse analysis; magnesium alloy **DOI**: 10.3969/j.issn.1674-6457.2014.06.005

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## 1 Introduction

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The magnesium alloy is a structure material which is applied widely for different mechanical sand electrical productions because of its higher ration of strength and density. The hot plastic forming is very important technology for manufacturing magnesium parts, therefore the research on numerical simulation of hot plastic forming process and prediction of the microstructure evolution have important significance to improve the performance of magnesium alloy products. A lot of numerical simulation results show that the constitutive relation and the constitutive parameters, which used for 文章编号:1674-6457(2014)06-0040-06

numerical calculation, have significant influence on the simulated results, specially the predicted microstructure results. Traditional method of measuring the material parameters is compression test and metallurgical observation of the deformed specimen and the data treatment by the volume average or area average method. However the treated data is not proper for numerical simulation of microstructure evolution because the specimen size is much larger than the grain size. For this reason, the parameter identification for magnesium alloy is the emphasis in this paper.

In order to realize prediction of microstructure during hot working, a thermal visco-plastic model considering dynamic recrystallization (VPDR) is developed firstly

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**Biography**: JIN Quan-lin(1944—), male, Beijing China, Ph. D., Professor, Main research filed: plastic mechanics, metal plastic forming, numerical simulation of plastic forming process and microstructure prediction.

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by Jin<sup>[1]</sup> and then improved by Qu et al recently<sup>[2]</sup>. The constitutive model includes more material parameters. It is very difficult to measure these parameters accurately by means of the existing conventional testing method because of complexity of the hot forging process. The inverse analysis technique may be a way to solve the problem. Its principle is to optimize the material parameters for minimizing a particular norm of the difference between the calculated values and the experimental results.

As well know the classical local optimization methods based on local differential<sup>[3-4]</sup> may be failing for the complex problem with non-convex objective function, so the global optimization algorithm has to be developed. The global algorithm includes both determinate approach and stochastic approach. The existing determinate algorithms use some mathematic characters of objective function, so its application is limited. The stochastic approach is a global search method. Most of them are based on evolutionary strategies mainly, such as the shuffled complex evolution method <sup>[5]</sup>, the realcoded genetic algorithm<sup>[6]</sup>. Considering slower convergence of the stochastic algorithm, most researchers think that only hybrid algorithms can lead to really interesting performance.

A hybrid evolutionary method is constructed to identify the parameters of VPDR in this paper. The algorithm includes the genetic algorithm with multiple crossover operators and L–M algorithm. A special numerical program for parameter identification of VPDR has been made through combination of above hybrid algorithm and a 2D program for simulating hot forging processing.

A set of satisfactory material parameters for AZ31D magnesium alloy is obtained respectively by the proposed inverse analysis numerical procedure. The set material parameters is validated through back extrusion experiment of a magnesium alloy AZ31D part and its numerical simulation. In any case, the parameter identification for macro-micro coupled hot plastic constitutive relation is very complex and difficult project <sup>[11-12]</sup>, where are many problems on theory and algorithm to be studied.

## 2 Thermal visco-plastic model

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The VPDR is developed firstly by Jin <sup>[1]</sup>, and improved by Qu and et al. <sup>[2]</sup> recently. This model emphasizes dynamic recrystallization evolution and its coupling with macroscopic deformation through dividing the recrystallized region and un-recrystallized region. In the model the microstructure evolution is described by the volume fraction of recrystallization X, un-recrystallized grain size  $D_1$ , recrystallized grain size  $D_2$ , the average grain size D and the maximum difference of grain size  $D_{dis}$ , where the subscript 1 and 2 denotes the variable in the unrecrystallized region and in the recrystallized region, respectively. In the model there are 19 material parameters to be identified:  $A_1$ ,  $A_2$ ,  $C_1$ ,  $C_2$ ,  $P_1$ ,  $P_2$ , Q, n,  $M_0$ ,  $Q_M$ ,  $a\gamma_0$ ,  $F_{10}$ ,  $F_{20}$ ,  $Q_F$ ,  $l_i/l_d$ , C, P,  $\beta_0$ ,  $Q_\beta$ .

## **3** Objective function

Objective function. Set the parameter vector K, upper limit vector U and lower limit vector L.

$$\begin{split} \boldsymbol{K}^{\mathrm{T}} &= \{A_{1}, A_{2}, C_{1}, C_{2}, P_{1}, P_{2}, Q, n, M_{0}, Q_{M}, a\gamma_{0}, \\ F_{10}, F_{20}, Q_{F}, l_{i}/l_{d}, C, P, \beta_{0}, Q_{\beta}\}, \ L \leq K_{i} \leq U_{i} \ (1) \end{split}$$
The objective function can be expressed as :

$$\phi(\mathbf{K}) = \frac{1}{2} \sum_{i=1}^{4} \sqrt{\omega_i} \| \mathbf{r}_i \|_2^2 = \frac{1}{2} \mathbf{R}(\mathbf{K})^{\mathrm{T}} \mathbf{R}(\mathbf{k}), \mathbf{r}_1 = \mathbf{F} - \mathbf{F}^*, \mathbf{r}_2 = \mathbf{D} - \mathbf{D}^*, \mathbf{r}_3 = \mathbf{X} - \mathbf{X}^*, \mathbf{r}_4 = \mathbf{D}_{\mathrm{dis}} - \mathbf{D}_{\mathrm{dis}}^*$$
(2)

Where  $\omega_i$  (i = 1, 2, 3, 4) are the weights of residual vector  $\mathbf{r}_i$  (i = 1, 2, 3, 4). In above equations,  $\mathbf{F}^*$  represents measured loads at different time points,  $\mathbf{X}^*$ ,  $\mathbf{D}^*$ ,  $\mathbf{D}^*_{\text{dis}}$  represent the measured values of the recrystallized volume fractions, the average grain sizes and the maximum grain size differences.  $\mathbf{F}, \mathbf{X}, \mathbf{D}, \mathbf{D}_{\text{ids}}$  are the related calculated values. Let  $\omega_1 = 1$  and  $\omega_2$ ,  $\omega_3$ ,  $\omega_4$  can be determined by the relative magnitude of  $\mathbf{r}_i$  (i = 1, 2, 3, 4).

**Calculation of objective function.** The experimental results in this paper are from the compression test of cylindrical specimen under constant temperature and constant velocity<sup>[7]</sup>. The experimental results include the load – displacement curve, distribution of average grain size, volume fraction of dynamic recrystallization, and maximum difference of grain size. The related calculated results are from numerical simulation of the upsetting test by using a special version of 2D rigid –plastic finite element program with the VPDR model. **Characteristics of objective function.** A lot of numerical calculation show that the objective function has the following characteristics: (1) The objective function is non–convex. (2) The parameter space for numerically feasible solution is not clear. (3) Huge compute resource may be need for calculating the objective function. (4) The parameter sensitivity of the objective function is poor.

## 4 Global optimization agorithm

Considering above characters of the objective function, a hybrid evolutionary method (HEM) is designed. This is a global optimization strategy, which is a combination of the genetic algorithm (GA)<sup>[6]</sup>. Levenberg-Marquardt (L - M) algorithm<sup>[3]</sup>, augmented Gauss – Newton (NL2SNO) algorithm<sup>[4]</sup> with the flexible tolerance method<sup>[8]</sup> and the concept of complex<sup>[9]</sup>. Because of its global search ability and slower convergence, GA in HEM is only for finding a good initial value of the solution, and then using L-M algorithm and NL2SNO refine gradually the obtained the solution. When the solution obtained by L-M algorithm and NL2SNO algorithm is numerically infeasible, the flexible tolerance method is used in order to find a numerically feasible solution to replace the numerically infeasible solution. The solution obtained by NL2SNO algorithm as a child of GA population turn to GA until the stop criterion is reached.

**Real-coded genetic algorithm** (**RGA**). As a global optimization technique, RGA includes production operator, crossover operator and mutation operator. Here a real number vector (parameter vector) is regarded as a representation of problem.

Generation of initial population. Considering varia-

tion interval of some parameters may be very large the method of population initialization is as follows.

$$K_{i} = \begin{cases} 10^{\lg L_{i} + \delta(\lg U_{i} - \lg L^{i})} & U_{i}/L_{i} \ge 100\\ L_{i} + \delta(U_{i} - L_{i}) & U_{i}/L_{i} \le 100 \end{cases}$$
(3)

**Mutation operator.** No – uniform mutation operator (Herrera et al., 1998) is a successful mutation operator, which makes a uniform search in the population at initial stage and very locally at a later stage. However the value from operator is biased towards the points with larger absolute value. In order to make the mutation operator search parameter space thoroughly as possible, the following hybrid no–uniform mutation operator is proposed.

$$\lg(K_i/K_{0i}) = \begin{cases} \beta \, \lg \, (U_i/K_{0i}) & U_i/L_i \ge 100, \delta \ge 0.5 \\ \beta \, \lg \, (L_i/K_{0i}), & U_i/L_i \ge 100, \delta < 0.5 \end{cases}$$
(4)

$$K_{i} = \begin{cases} K_{0i} + \beta (U_{i} - K_{0i}) & U_{i} / L_{i} < 100, \ \delta \ge 0.5 \\ K_{0i} + \beta (K_{0i} - L_{i}) & U_{i} / L_{i} < 100, \ \delta < 0.5 \end{cases}$$
$$\lg (1 - \beta) = (1 - t / g_{max})^{5} \lg \alpha$$

In above equations,  $K_{0i}$  is the gene of parent, t and  $g_{\max}$  is the current generation number and set maximum generation number respectively.

**Crossover operator.** The crossover operator is the core of genetic algorithm. The above method used for the initial population and the mutation operator is also applied to the crossover operator.

Considering advantages of different crossover operators, a hybrid crossover operator is designed. The hybrid crossover operator combines SBX crossover operator, Random simplex crossover operator, FCB crossover operator, arithmetical crossover operator with one dimensional golden section local search algorithm. By using the concept of complex shuffling, several parents are selected from population P as a complex A, and then the crossover operator and mutation operator are applied in a complex; the generated children from A are shuffled in P at last.

Generation-alteration model (GAM). GAM determines how to choose pairs of parents for generating children by the crossover operators and the mutation operators and how to select parents surviving in the next generation. Here GAM role is to search thoroughly parameter space, because GA is used only for finding a good initial solution of classic optimization method. After arraying N chromosomes in order of decreasing objective function, the selection probability of chromosome i is as:

$$\rho_i = i/[(N+1)N] \quad (i=1,\cdots,N) \tag{5}$$

After this, the parents used for mutation and crossover are selected according to Roulette rule.

**Classic optimization method.** Classic local optimization method is based on the following quadratic Taylor expansion of objective function  $\phi(\mathbf{K})$  at  $\mathbf{K}_{i}$ .

$$\phi(\mathbf{K}) = \phi(\mathbf{K}_{t}) + \mathbf{R}_{t}^{\mathrm{T}} \mathbf{J}_{t} \Delta \mathbf{K} + \frac{1}{2} \Delta \mathbf{K}^{\mathrm{T}} \mathbf{H}_{t} \Delta \mathbf{K}$$
(6)

Where  $\boldsymbol{R}_{t} = \boldsymbol{R}(\boldsymbol{K}_{t})$ ,  $\boldsymbol{J}_{t} = \nabla \boldsymbol{R}_{t}$ ,  $\boldsymbol{H} = \boldsymbol{J}_{t}^{\mathrm{T}} \boldsymbol{J} + \boldsymbol{R}_{t} \nabla^{2} \boldsymbol{R}_{t}$ . The Only  $\mathbf{J}_{\perp}^{\mathrm{T}}\mathbf{J}_{\perp}$  is considered in L-M algorithm based on Gauss-Newton model  $q_1^{G}(K_{t+1})$  and the trust region technology. However, the error due to ignoring  $\mathbf{R}_1 \nabla^2 \mathbf{R}_1$  may be larger because of nonlinear interaction among parameters and large residuals  $R_{\perp}$ . NL2SNO based on augmented Gauss-Newton model  $q_t^s(\mathbf{K}_{t+1})$  is just for dealing with the case because of including  $\mathbf{R}_{1} \nabla^{2} \mathbf{R}_{1}$ . On other hand, more tests show that predicted  $\phi(\mathbf{K}_{i+1})$  by  $q_i^{G}(\mathbf{K}_{i+1})$  is often better than that by  $q_i^{S}$  $(\mathbf{K}_{1+1})$  for small time steps. For this reason, the L-M algorithm is first is started up to refine the initial solution from GA until  $\Delta K_{t}$  is enough small. After this, NL2SNO algorithm runs to make a last few refining iteration. When some solutions given by L - M and NL2SNO may be numerically infeasible, the feasible tolerance method is adopted for producing a numerically feasible solution. The feasible tolerance method is not only able to avoid that the iteration process is trapped by minor optimum but also in dependent on derivative of objective function.

In L-M algorithm a set of constraints is added into the objective function by weighted penalty function. Let denote the solution obtained by L-M algorithm, then the set of constraints is: if  $\overline{K}_i < L_i$  then  $K_i = L_i$ , if  $L_i < \overline{K}_i < U_i$ , then  $K_i = \overline{K}_i$ , if  $\overline{K}_i > U_i$ , then  $K_i = U_i$ .

Feasibility enforcement operator (FEO). A solu-

tion is defined as an infeasible solution when the calculation of objective function is interrupted, or the calculated results do not agree obviously with the general knowledge. In the population initialization phase and early search phase by crossover operator and mutation operator, the generated solution may be infeasible with high probability. A feasibility enforcement operator is designed to make an infeasible solution feasible as possible. The feasibility enforcement operator means the prescriptive variation interval of some variables ( such as flow stress, grain size and etc. ) according to experience and the experimental data.

## 5 Parameter identification for magnesium alloy AZ31D

A software for parameter identification is developed by combination of the global optimization algorithm and the rigid–plastic FEM program. The parameter identification of VPDR for AZ31D magnesium alloy is carried out by using the software.

In order to provide input experimental data for parameter identification as initial value of the iteration process, a set of compression of cylindrical specimen of magnesium alloy AZ31D is carried out under different temperature, different strain rate and different strain. The distribution and its variation of grain size in the compressed specimen with deformation condition are observed experimentally. The conferences<sup>[7,10]</sup> give the detailed experimental results. The main experimental data used for parameter identification include the stress -strain curves obtained for the compression tests, grain size evolution under different deformation conditions, the load-displacement curves of the compression tests. In the parameter identification calculation, the stressstrain data used to estimate the parameter iteration initial value and selected parameter range, the load data and grain data are used to objective function calculation for evaluate the difference between the selected parameters and ideal parameters. The important input data for parameter identification of AZ31D is listed in Table 1, where Li and Ui are the lower limit and upper

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limit of the parameter range respectively. The identified parameters for AZ31D are listed in Table 2.

#### Table 1 Input dta for prameter ientification of AZ31D 表 1 AZ31D 参数识别的输入数据

Parameter	Unit	Initial value	$L_i$	$U_i$
$A_{\rm un}$	N/mm <sup>2</sup>	19.23	10	400
$c_{\rm un}$		0.000 479	0.0001	0.4
$p_{ m un}$		0.8674	0.1	0.8
$A_{\rm re}$	N/mm <sup>2</sup>	19.23	10	100
$c_{ m re}$		0.009 31	0.0001	0.4
$p_{ m re}$		0.5678	0.1	0.8
Q	kJ/mol	86 230	50	300
$F_{\rm un0}$	$mm^4/(N^2\boldsymbol{\cdot}s)$	28.53	$3 \times 10^{-2}$	$3 \times 10^{4}$
${F}_{ m re0}$	$mm^4/(N^2 \cdot s)$	27	$3 \times 10^{-2}$	$3 \times 10^{4}$
$Q_F$	kJ/mol	114.64	45	400
$a \boldsymbol{\gamma}_0$	J/mm <sup>2</sup>	3.7×10 <sup>-5</sup>	9×10 <sup>-8</sup>	$14 \times 10^{-4}$
$L_i/L_d$		3.003	2	20
$M_0$	$mm^2/(J \cdot s)$	2.63×10 <sup>11</sup>	$2 \times 10^{9}$	$2 \times 10^{14}$
$Q_{\rm M}$	kJ/mol	94.273	50	300
$b_0$	Jmm/N <sup>2</sup>	0.014	$1.4 \times 10^{-5}$	140
$Q_{ m b}$	kJ/mol	73.775	50	200
n'		0.538	0.2	1
c	mm	365.18	1	1000
p		-1.095	-2.8	-0.25

#### Table 2 Identified parameters for AZ31D 表 2 通过参数识别得出的 AZ31D 参数

Parameter	Unit	Parameter value
$A_{un}$	N/mm <sup>2</sup>	24.11
$c_{\rm un}$		0.005
$p_{ m un}$		0.6326
$A_{ m re}$	N/mm <sup>2</sup>	24.11
$c_{ m re}$		0.0096
$p_{ m re}$		0.5746
$aoldsymbol{\gamma}_0$	J/mm <sup>2</sup>	9.96×10 <sup>-5</sup>
c	Mm	347.4
p		-1.086
n'		0.52
$L_i/L_d$		4.941
Q	J∕ mol	89 000
$Q_{ m b}$	J/mol	72 248.5
$M_0$	$mm^2/(J \cdot s)$	$2.23 \times 10^{11}$
$b_0$	$J \cdot mm/N^2$	0.092 41
$Q_F$	J/mol	129 872
$F_{\rm un0}$	$mm^4/(N^2 \cdot s)$	33.32
${F}_{ m re0}$	$mm^4/(N^2 \cdot s)$	31.832
$Q_{\rm M}$	J/mol	92 130

In order to validate the identified results, a back extrusion process of an AZ31D part is carried out. The Fig. 1 gives the formed part and the photos of the part profile where grain distribution can be measured. The back extrusion process of an AZ31D part is simulated numerically by using the obtained material parameters. A comparison of the calculated results and the experimental results on dynamic recrystallization and grain size evolution are shown in Fig. 1–2 and in Table 3. It is can be seen that the calculated results agree very well with the experimental observation.

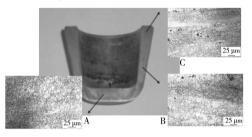
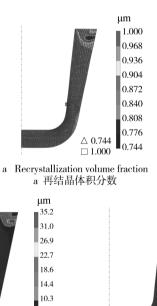


Fig. 1 Experimental results on microstructure distribution of formed AZ31D part

图1 成形零件微观组织分布的实验结果



46.3 30.9 15.4 0.00 c Maximum difference of grain size

μm

124

108

92.7

77.2

61.8

c 最大晶粒尺寸差

Fig. 2 Simulated results on microstructure distribution of formed AZ31D part (step 124)

6.14

2.00

 $\triangle 2.00$ 

□ 35.2

b Average grain size b 平均晶粒尺寸

图 2 成形零件微观组织分布的数值模拟结果

表 3 AZ31D 微观组织的模拟结果与实验结果的对比							
Region No	А	В	С				
Recrystallized	Experimental	3~4	3~4	10			
grain size $D_2/\mu m$	Simulated	3~4	3~5	2~4			
Un-recrystallized	Experimental	30	50	200			
grain size $D_1/\mu m$	Simulated	$16 \sim 42$	$25\sim\!55$	$90 \sim 120$			
Average grain size	Experimental	5	5	30			
D∕ µm	Simulated	$2 \sim 6$	$2 \sim 6$	$14 \sim 30$			
Grain size difference	Experimental	25	45	110			
$D_{\rm is}/\mu{ m m}$	Simulated	$0 \sim 30$	$0 \sim 46$	93			
Recrystallized volume	Experimental	100	100	60			
receiystamized volume	Experimentai	100					

# Table 3 Comparison of simulated and experimental results on microstructure for AZ31D

## 6 Conclusion

In order to identify the material parameters of VPDR, an objective function with multiple objects is established, and a HEM including RG, L–M and  $NL_2SNO$ , is constructed. A special program for parameter identification is developed by a combining FEM and HEM. By using the program, the parameter identification for AZ31D magnesium alloy is carried out. A comparison between calculated results and experimental results on the back extrusion process of an AZ31D part shows that the parameter identification program and the identified material parameters are all reliable.

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### 镁合金 AZ31D 热粘塑性模型的参数识别

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摘要:通过联合实数编码遗传算法和一些经典的局部优化方法,构造了一种混合型的全局优化方法并用于开发一个专用的参数识别程序。使用这个专用程序进行了镁合金 AZ31D 的参数识别。 变形试验和数值模拟结果的比较显示,本参数识别方法及其所得到的材料参数都是可用的。所得 到的本构参数可用于镁合金零件热塑性成形的数值模拟。 关键词:参数识别;动态再结晶;反分析;镁合金